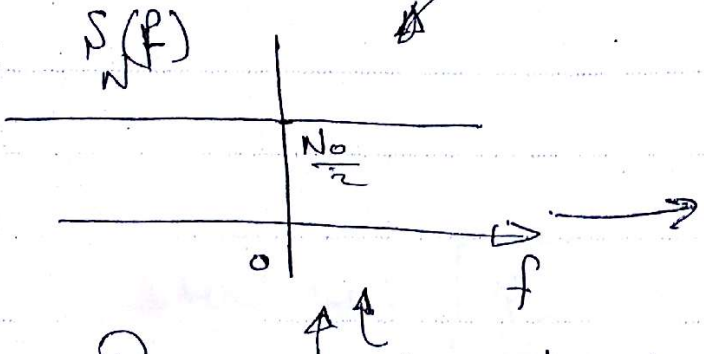
Center Share

Ch1 (Noise)

Noise: is any unwanted signal that disturb the transmission of signals.

وأهم تلك الأنواع هو الـ white noise والذي يتواجد في كل channel in Comm. system وتكون توزيع الـ white noise على كل الترددات f كما بالشكل

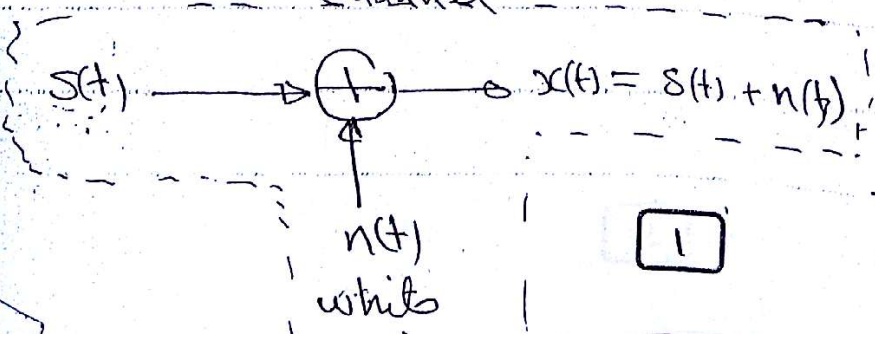


Power spectrum density of white noise (PSD)

Center Share

Additive noise channel

وتم تمثيلها بالشكل التالي حيث انه تم إضافة الـ white noise الى system او channel بسبب تأثير الـ noise على الـ channel



1

لذا سمي الـ white noise بـ additive white noise

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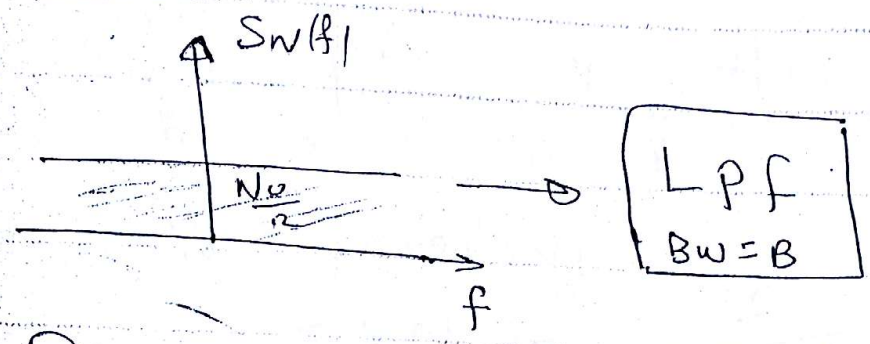
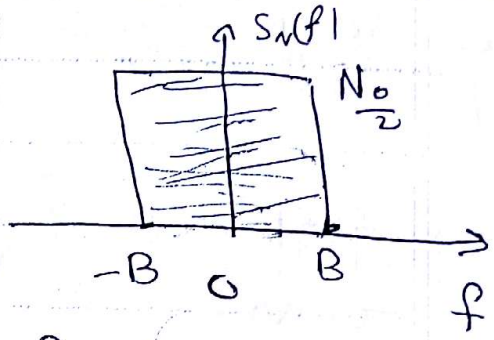
(I) (Ideal Low-Pass white noise)

Center Filter

الـ noise مرت على تقليل تأثير الـ power الموجود في الـ





$P_n = \infty$

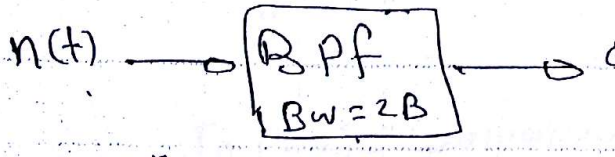
P_n = Area

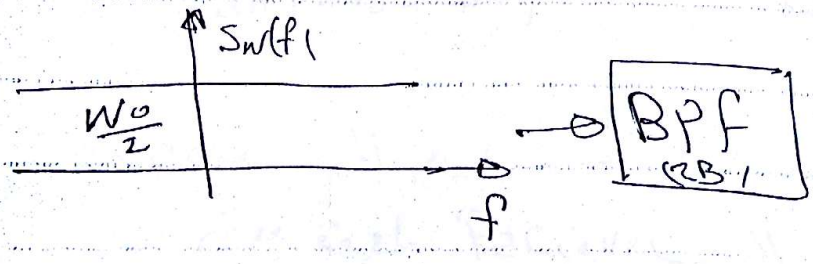
$= 2B \times \frac{N_0}{2} = N_0 B$

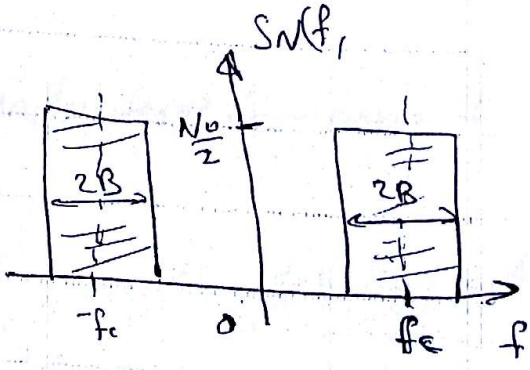
∴ after Lpf $\boxed{P_n = N_0 B < \infty}$

Center Sliding

(II) (Bandpass noise (Narrow banding noise))







$P_n = \infty$

$P_n = \frac{N_0}{2}[2B + 2B]$

$\boxed{P_n = 2N_0 W}$ $\boxed{B = W}$

$\boxed{P_n = 2N_0 B}$

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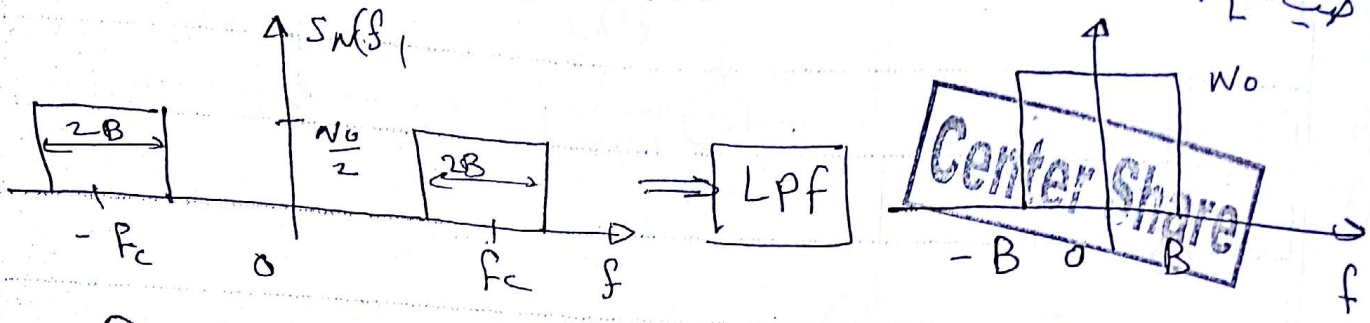
II (III) Ideally filtered narrow band noise

وهذا الشكل يهم بمرور الإشارة على الـ Rx في الـ DeModulation

حيث تم إدخال الـ Noise (Bpf) & Lpf كما بالشكل التالي



$$P_n = (2B+2B)\frac{N_0}{2} = 2N_0 w$$

$$P_n = 2BN_0 = 2N_0 w$$

ملحوظة ① عند التعامل مع الـ $n(t)$ في الـ Channel يتم التعامل عبر المعادلة التالي

Center Share

$$n(t) = n_I(t)\cos 2\pi f_c t - n_Q(t)\sin(2\pi f_c t)$$

In phase Component — Quadrature Component

ملحوظة ② أجزاء المقدر لتصرف منه حالات الـ noise عند دخل وخرج الـ Reciever لمعرفة مدى جودة الـ Rx وكم نستطيع أن نتعامل بين الأنواع المختلفة

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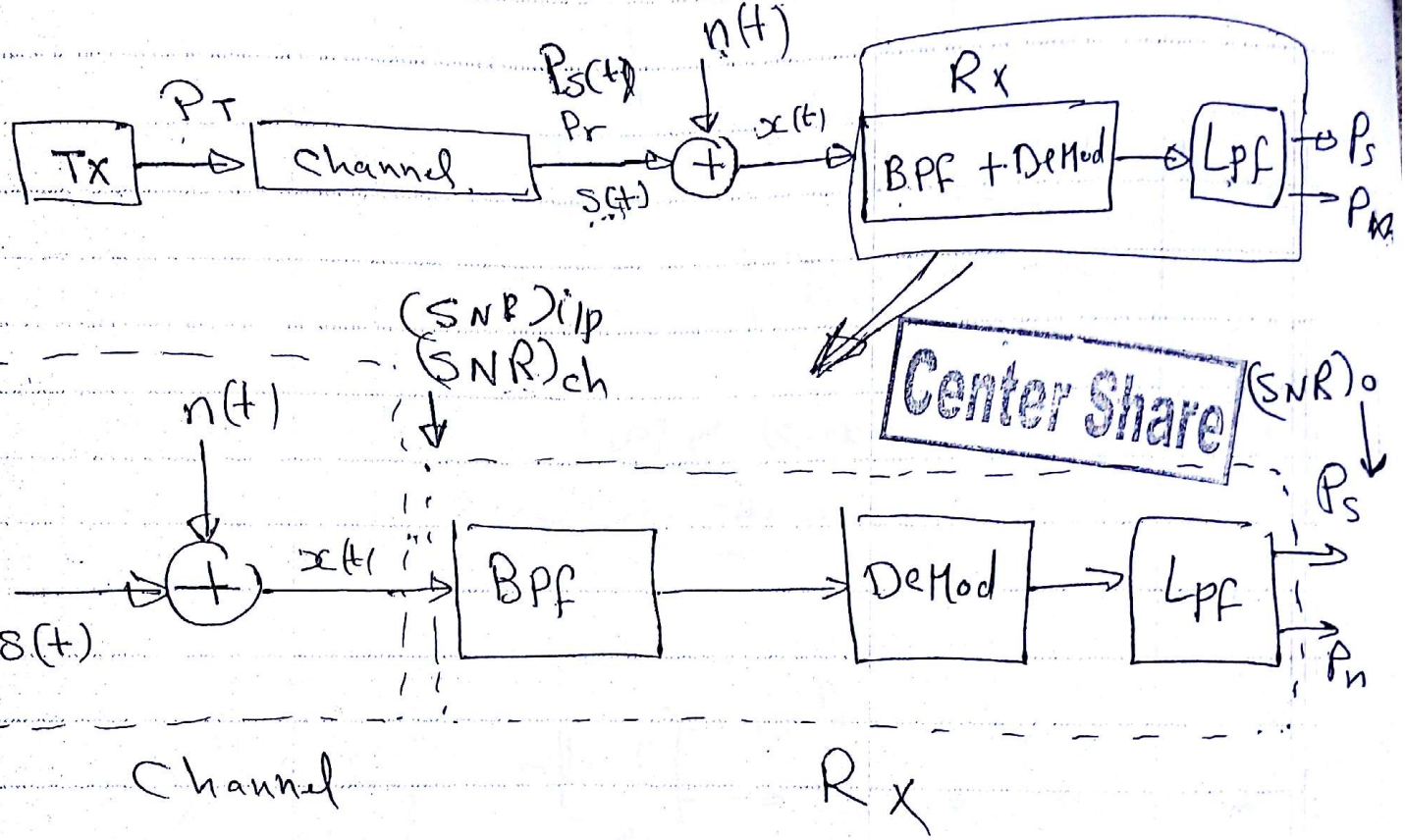
Noise in C.S



الصنف ثم أي نوع « AM - DSBSC - SSB - FM » هو

معرفه منه الـ $(SNR)_i$ والـ $(SNR)_o$

$(SNR)_o$ ↓ at Rx o/p

Center Share at Rx i/p

where

$$SNR = \frac{\text{average signal power}}{\text{average noise power}}$$

* ثم تم حساب الكفاءة بنظام Figure of Merit

Figure of Merit = $\frac{(SNR)_o}{(SNR)_{ch}}$ = FOM

الـ FOM كلما زادت كلما

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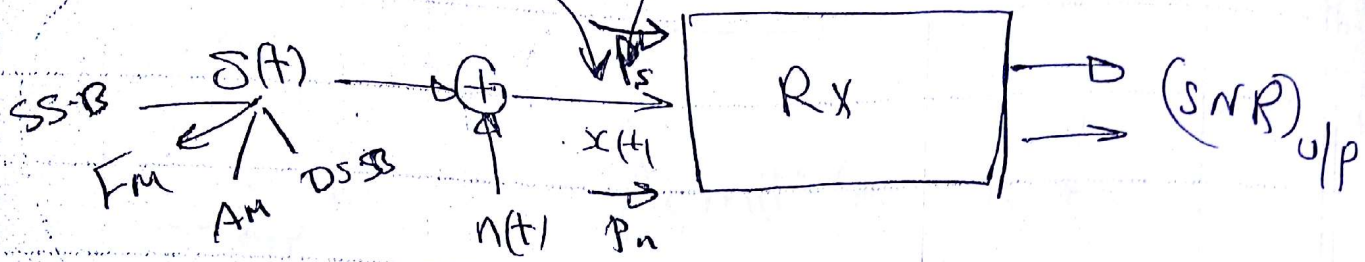
وعليه

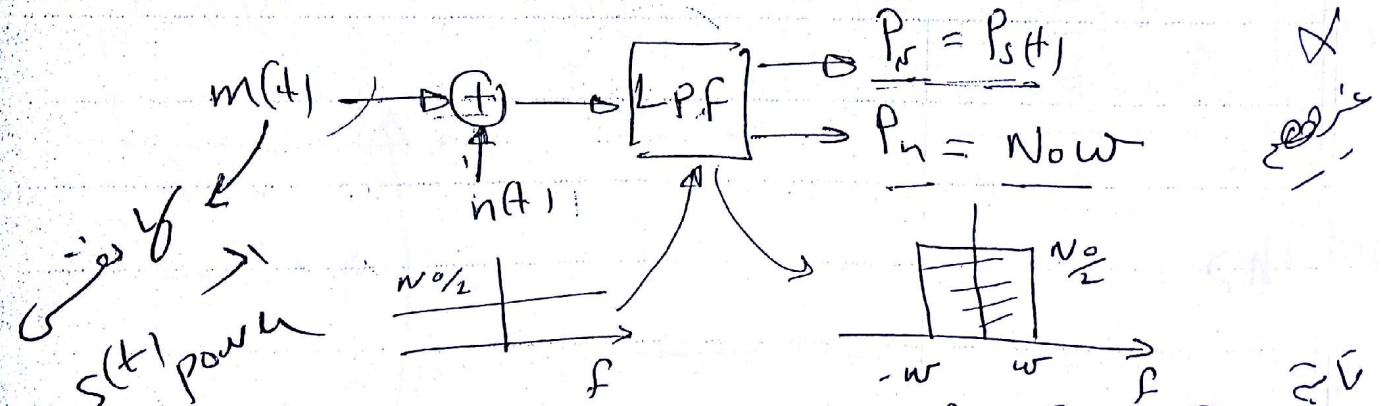
(I) $(SNR)_{ch} = (SNR)_{i/p}$



$$(SNR)_{ch} = \frac{P_s}{P_n} = \frac{P_{s(t)}}{P_n}$$

Center Share

وكما تم صياغة الـ $(SNR)_{ch}$ تم فرض أنه طالما الإشارة $m(t)$ لها نفس الـ Power الخاصة بـ $s(t)$ وتم إضافة الـ $n(t)$ عليها كما بالشكل التالي



$$\therefore (SNR)_{i/p} = (SNR)_{ch} = \frac{P_{s(t)}}{N_o w}$$

Center Share

$P_n = N_o w$

معادلة خاصة لكل الأنواع

والغرض العام سوف تتم إستخدامه مع جميع أنواع Mod وسوف نلاحظ أن الجزء الأهم والخاص هو الـ $P_{s(t)}$ والذي سوف تختلف بحسب نظام الإرسال

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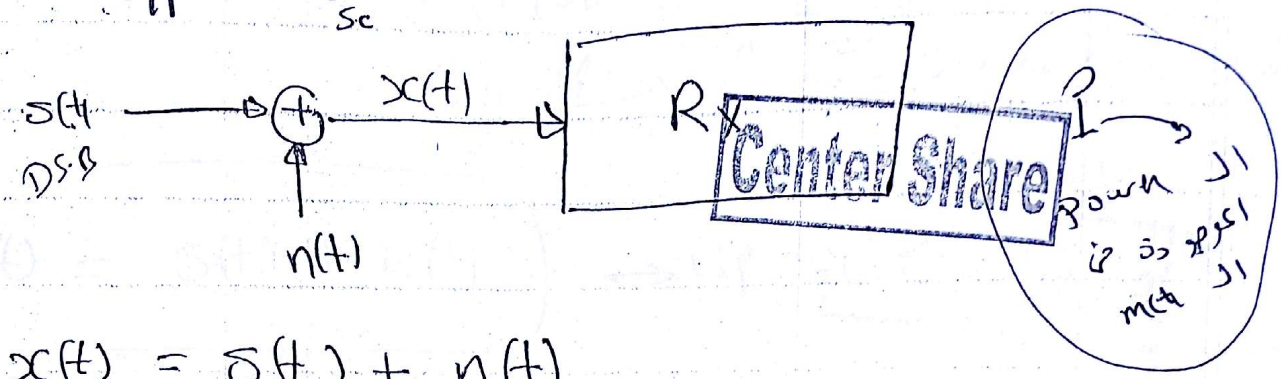
I Noise In DSBSC

(a) $(SNR)_{i/p}$

At (TX) o/p :- $s(t) = A_c m(t) \cos 2\pi f_c t$

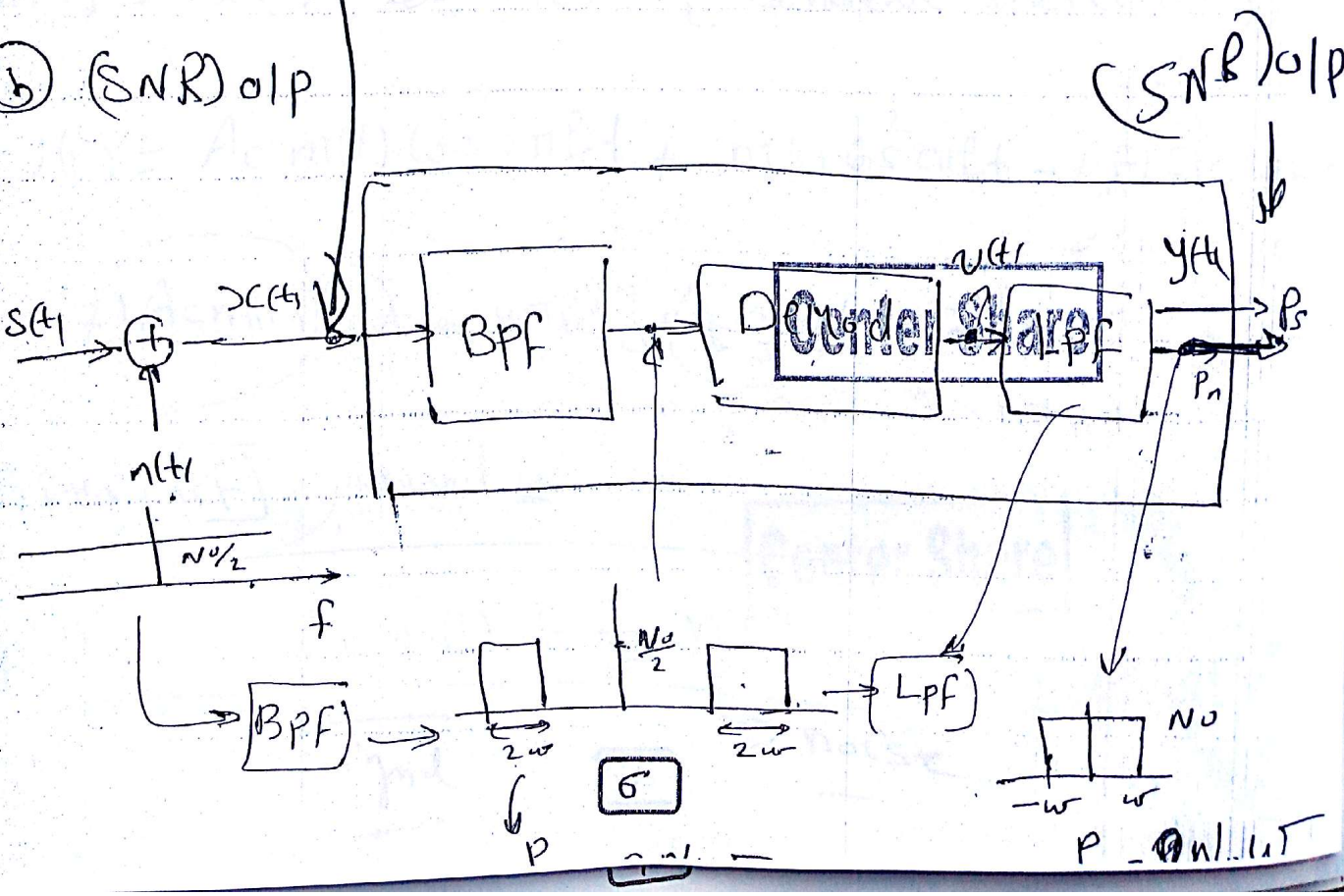
DSB SC



$x(t) = s(t) + n(t)$

$$(SNR)_{i/p} = \frac{P_{s(t)}}{P_n} = \frac{A_c^2 P/2}{N_0 W} = \frac{A_c^2 P}{2(N_0 W)} \quad (1)$$

(b) $(SNR)_{o/p}$



ا شكل الـ بعد وجدنا انه شكل الـ noise بعد انه عبر على
فلاتر الـ Bpf والـ Lpf تكون
الـ $P_{n_{o/p}} = 2N_0 w$ وهكذا هنا الرقم ليس مزعج لدينا لم
نأخذ في الاعتبار تأثير الـ DeMod (الكاشف) والذي سوف
نتعرف بالمعادلات كيف شكل الناتج

$$x(t) = s(t) + n(t)$$ → i/p of Rx Center Share

$$= A_c m(t) \cos 2\pi f_c t + [n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t]$$

After DeMod

$$v(t) = x(t) \cdot \cos 2\pi f_c t \quad [\text{Coherent Detection}]$$

$$v(t) = A_c m(t) \cos^2 2\pi f_c t + n_I(t) \cos^2 2\pi f_c t - n_Q(t) \sin 2\pi f_c t \cdot \cos 2\pi f_c t$$

$$= \frac{1}{2} A_c m(t) [1 + \cos 4\pi f_c t] + \frac{1}{2} n_I(t) [1 + \cos 4\pi f_c t] + \frac{1}{2} n_Q(t) \sin 4\pi f_c t$$

After [LPF] around 0

Center Share

$$y(t) = \frac{1}{2} A_c m(t) + \frac{1}{2} n_I(t)$$

signal → noise

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$$y(t) = \underbrace{\frac{1}{2} A_c m(t)}_{\text{signal}} + \underbrace{\frac{1}{2} n_I(t)}_{\text{noise}}$$

$$\left(P_o = \frac{1}{4} A_c^2 P\right) + \left(\frac{1}{4}(P_n)\right)$$

. noise

ثم حساب مقدار القدرة

$= 2N_o W$

$$\therefore P_o = \frac{1}{4} A_c^2 P \longrightarrow \text{o/p signal}$$

$$P_{n_o} = \left(\frac{1}{4}\right)(2N_o W) = \left(\frac{N_o W}{2}\right) \longrightarrow \text{o/p Noise}$$

Center Share

$$\therefore (SNR)_o = \frac{\frac{1}{4} A_c^2 P}{\frac{N_o W}{2}} = \frac{A_c^2 P}{2 N_o W} \longrightarrow (2)$$

$$(SNR)_{ch} = \frac{A_c^2 P}{2 N_o W} \longrightarrow \text{from (1)}$$

$$\therefore \frac{(SNR)_{o/p}}{(SNR)_{i/p}} = 1 \leq FOM$$

وهذا يسمى System لا يضيف ولا يقلل من الـ Noise

Center Share

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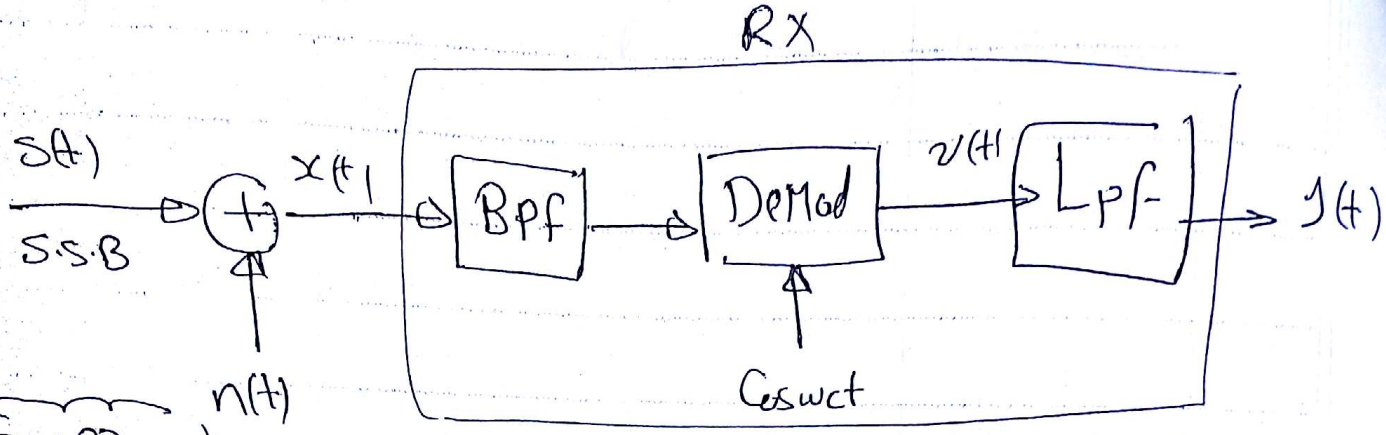
[2] Noise in S.S.B Modulation

نفس الطريقة مع اختلاف معادلات الـ S.S.B



$(SNR)_{ch}$

$$S(t) = \frac{A_c}{2} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t$$

$\hat{m}(t)$ → Hilbert transform

$$P_{s(t)} = \frac{A_c^2}{8} P + \frac{A_c^2}{8} P = \frac{A_c^2}{4} P$$

Center Share

$$P_{n_{i/p}} = N_0 w$$

$$\therefore (SNR)_c = \frac{A_c^2 P/4}{N_0 w} = \frac{A_c^2 P}{4(N_0 w)} \quad (1)$$

$(SNR)_{o/p}$

assume Lower Side Band

Center Share

$$x(t) = s(t) + n(t)$$

$$x(t) = s(t) + n_s(t) \cos 2\pi\left(f_c - \frac{w}{2}\right) t - n_c(t) \sin 2\pi\left(f_c - \frac{w}{2}\right) t$$

after demodulation

المستقر

$$v(t) = x(t) \cdot \cos 2\pi f_c t \rightarrow \text{Coherent detection}$$

المركز

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After LPF

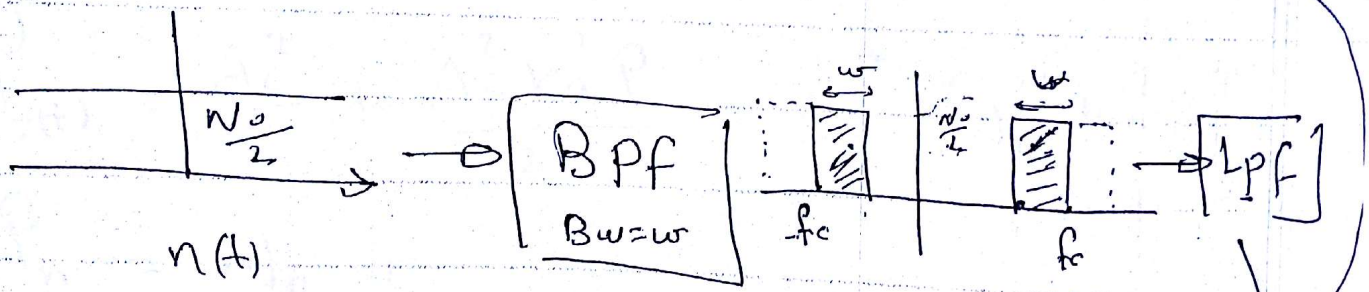
$$y(t) = \frac{A_c}{4} m(t) + \frac{1}{2}\left[n_I(t)\cos\omega_c t - n_Q(t)\sin\omega_c t\right]$$

o/p signal — o/p noise

$$P_{S_{o/p}} = \frac{A_c^2 P}{16}$$

$$P_{n_{o/p}} = \left(\frac{1}{4}\right)(\quad) \quad \text{مع العلم}$$

Center Share



$$P_{n_{o/p}} = \left(\frac{1}{4}\right)(N_0 w) = \frac{N_0 w}{4}$$

$$(SNR)_{o/p} = \frac{A_c^2 P/16}{N_0 w/4} = \frac{A_c^2 P}{4 N_0 w}$$

Center Share

$$(SNR)_{i/p} = \frac{A_c^2 P}{4 N_0 w} = (SNR)_{o/p}$$

10

FOM = 1

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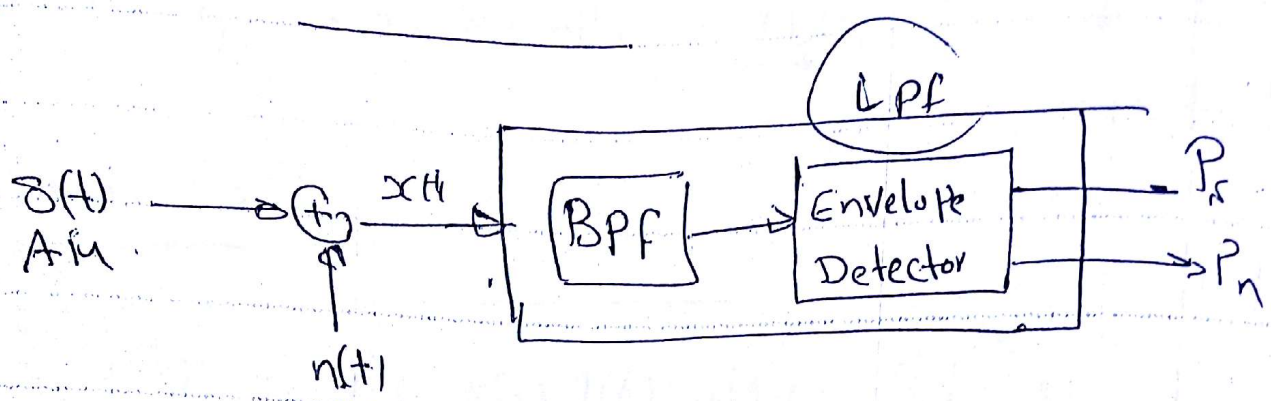
(3) AM - T.C



(1) (SNR)i/p

Center Share

$$s(t) = A_c[1 + k_a m(t)] \cos \omega_c t$$

$$= A_c \cos \omega_c t + A_c k_a m(t) \cos \omega_c t$$

$$P_{s(t)} = \frac{A_c^2}{2} + \frac{A_c^2 k_a^2 P}{2} = \frac{A_c^2}{2}[1 + k_a^2 P]$$

$$P_n = N_0 \omega$$

$$\therefore (SNR)_{ch} = \frac{A_c^2[1 + k_a^2 P]}{2(N_0 \omega)} \rightarrow (1)$$

Center Share

(3) (SNR) o/p

$$\because x(t) = s(t) + n(t)$$

$$x(t) = [A_c + A_c k_a m(t) + n_I(t)] \cos \omega_c t - n_Q(t) \sin \omega_c t$$

$$= (A_c \cos \omega_c t) + (A_c k_a m(t) \cos \omega_c t) + [n_I(t) \cos \omega_c t - n_Q(t) \sin \omega_c t]$$

$$\because n(t) = n_I(t) \cos \omega_c t - n_Q(t) \sin \omega_c t$$

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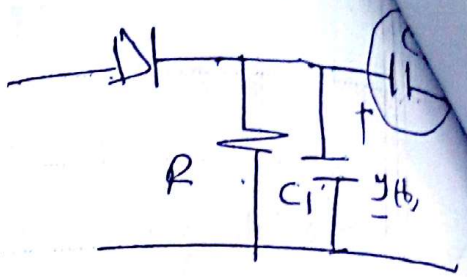
After envelope Detector

$$y(t) \cong \underbrace{A_c}_{DC} + A_c k_a m(t) + n_I(t)$$



After C_2

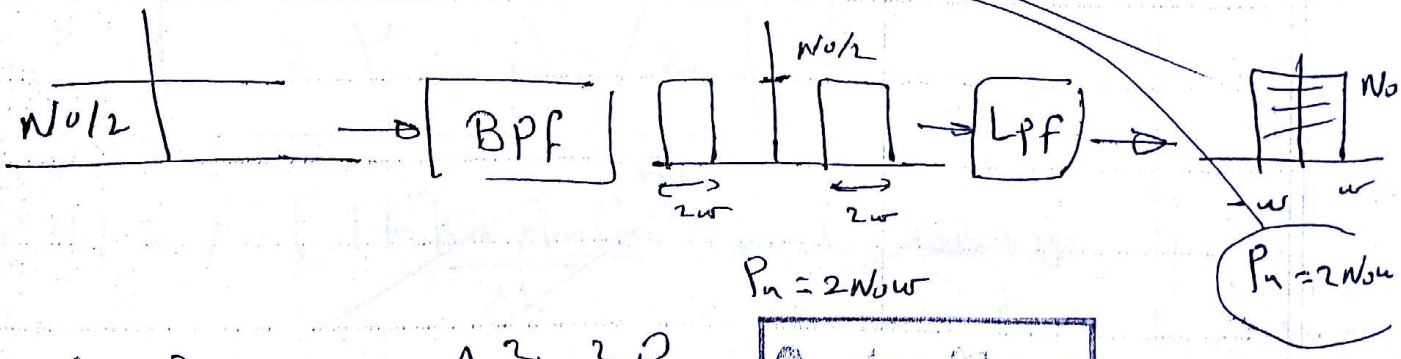
$$y(t) = A_c k_a m(t) + n_I(t)$$

(signal) (noise)

Center Share

$$P_{s_{o/p}} = A_c^2 k_a^2 P$$

$$P_{n_{o/p}} = (1)(2 N_0 W)$$ ← From filter



Center Share

$$\therefore (SNR)_{o/p} = \frac{A_c^2 k_a^2 P}{2 N_0 W}$$

$$(SNR)_{ch_{i/p}} = \frac{A_c^2 [1 + k_a^2 P]}{2 N_0 W}$$

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$\therefore$ Figure of Merit = FOM = $\frac{Ka^2 P}{1 + Ka^2 P} < 1$

* لاحظ ان $(SNR)_{o/p}$ أقل من $(SNR)_{i/p}$ وذلك بسبب وجود ال Carrier الذي يستهلك جزء كبير من ال Power وهو لا يحمل أي معلومات

Center Share

Example

Single tone Modulation

Consider $m(t) = A_m \cos 2\pi f_m t$ (cos أو sin)

Find FOM for AM system

(Solution)

$\therefore m(t) = A_m \cos 2\pi f_m t$

$\therefore P = A_m^2 / 2$

Center Share

$s(t) = A_c [1 + K_a A_m \cos 2\pi f_m t] \cos 2\pi f_c t$

$= A_c [1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t$

$K_a A_m = \mu$

$\therefore FOM = \frac{(SNR)_o}{SNR_i} = \frac{K_a^2 P}{1 + K_a^2 P} = \frac{K_a^2 A_m^2/2}{1 + K_a^2 \frac{A_m^2}{2}} = \frac{\mu^2/2}{1 + \frac{\mu^2}{2}}$

$FOM = \mu^2$

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ملخص القوانين الباقية

Center Share

System	$(SNR)_{i/p}$	$(SNR)_{o/p}$	η
DSB S.C	$\frac{A_c^2 P}{2N_0 W}$	$\frac{A_c^2 P}{2N_0 W}$	1
S.S.B	$\frac{A_c^2 P}{4N_0 W}$	$\frac{A_c^2 P}{4N_0 W}$	1
AM	$\frac{A_c^2 [1+k_a^2 P]}{2N_0 W}$	$\frac{A_c^2 k_a^2 P}{2N_0 W}$	$\frac{k_a^2 P}{1+k_a^2 P} < 1$

Center share

	o/p noise معدل الـ	P_n (filter)	$P_{n_{o/p}}$ Total
DSB	$\frac{1}{4} = (\frac{1}{2})^2$	$2N_0 W$	$\frac{N_0 W}{2}$
S.S.B	$\frac{1}{4}$	$N_0 W$	$\frac{N_0 W}{4}$
AM	1	$2N_0 W$	$2N_0 W$

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Sheet 1 (Noise)

1-a A DSB-SC modulared signal is transmitted over a noisy channel, with the power spectral density of the noise being as shown in Figure P2.46. The message bandwidth is 4 kHz and the carrier frequency is 200 kHz. Assuming that the average power of the modulated wave is 10 watts, determine the output signal-to-noise ratio of the receiver.

Center Share

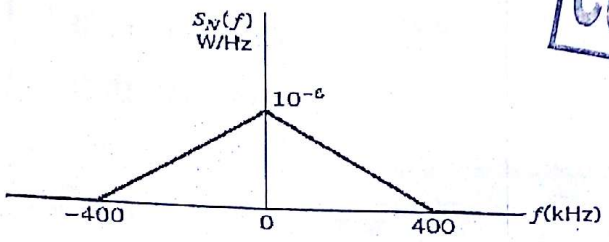

FIGURE P2.46

1-b Repeat for SSB – SC with upper side

==========

2- A certain communication channel is characterized by 90-dB attenuation and additive white noise with power-spectral density of $\frac{N_0}{2} = 0.5 \times 10^{-14}$ W/Hz. The bandwidth of the message signal is 1.5 MHz and its amplitude is uniformly distributed in the interval $[-1, 1]$. If we require that the SNR after demodulation be 30 dB, in each of the following cases find the necessary transmitter power.

1. USSB modulation.
2. Conventional AM with a modulation index of 0.5.
3. DSB-SC modulation.

Center Share PJ=1/3 watt

==========

3- Let a message signal $m(t)$ be transmitted using single-sideband modulation. The power spectral density of $m(t)$ is

$$S_M(f) = \begin{cases} a\frac{|f|}{W}, & |f| \le W \\ 0, & \text{otherwise} \end{cases}$$

where a and W are constants. White Gaussian noise of zero mean and power spectral density $N_0/2$ is added to the SSB modulated wave at the receiver input. Find an expression for the output signal-to-noise ratio of the receiver.

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مادة : هندسة الاتصالات

الفرقة الثانية – اتصالات

Center Share

4- Report

In a broadcasting communication system the transmitter power is 40 KW, the channel attenuation is 80 dB, and the noise power-spectral density is 10^{-10} W/Hz. The message signal has a bandwidth of 10^4 Hz.

a- Find the output SNR if the modulation is DSB.

b- Find the output SNR if the modulation is SSB.

c- Find the output SNR if the modulation is conventional AM with a modulation index of 0.85 and normalized message power of 0.2.

Center Share

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sheet ①

① @ $W = 4KH_3$ $f_c = 200KHz$ $P_{s(t)} = 10 watts$

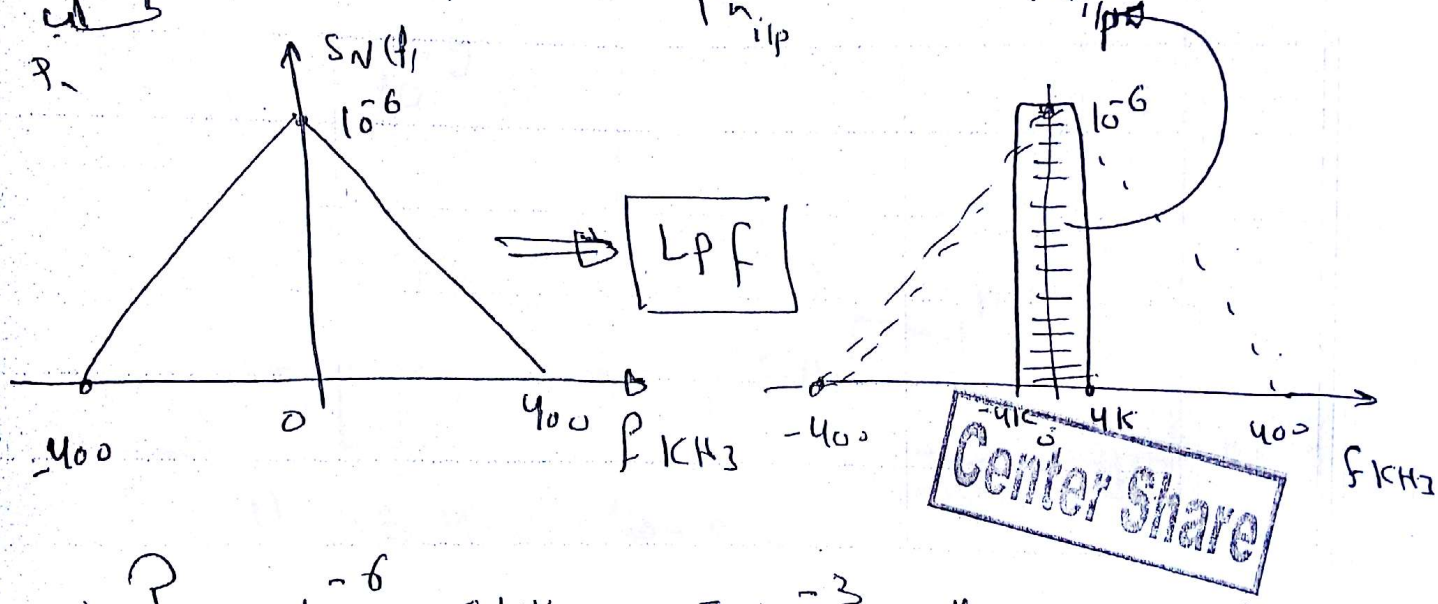
$(SNR)_{o/p} = ??$

Center Share

@ for D.S.B.SC SOL

$\therefore (SNR)_{i/p} = (SNR)_{o/p}$

$\therefore (SNR)_{i/p} = \frac{P_{s(t)}}{P_{n_{i/p}}} = \frac{10 watt}{P_{n_{i/p}}}$



Center Share

$\therefore P_n = 10^{-6} * 8KH_3 = 8 * 10^{-3} watt$

$\therefore (SNR)_{i/p} = \frac{10}{8*10^{-3}} = 1250 = SNR_o$ *

(b) For SS.B

$(SNR)_{i/p} = (SNR)_{o/p} = \frac{P_{s(t)}}{P_n}$

$\therefore P_{s(t)} = 10 watts$

$P_{n_{SS.B\ i/p}} = P_{n_{D.S.B\ i/p}} = 10^{-6} * 8K = 8 * 10^{-3} watt$ (1)

$\therefore (SNR)_{o/p} = \frac{10}{} = 1250$

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Q2

Attenuation = 90 dB

$\frac{N_0}{2} = 0.5 \times 10^{-14}$ w/Hz $\rightarrow N_0 = 10^{-14}$ w/Hz

$W = 1.5$ MHz

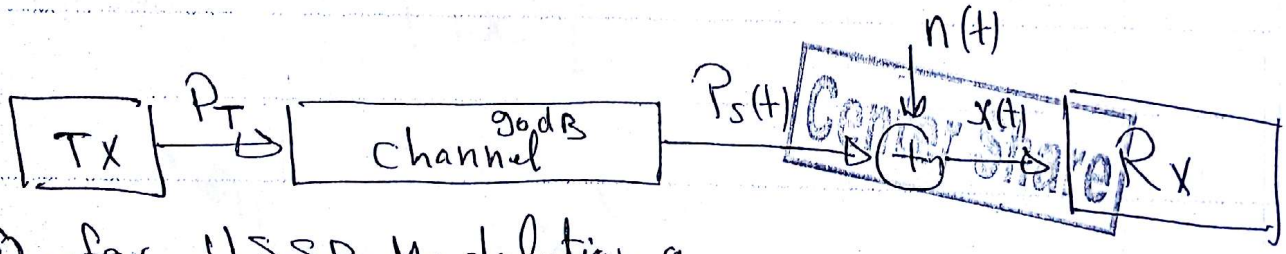
$(SNR)_{o/p} = 30$ dB $= 1000$

$P_T = ??$

Sol

Channel attenuation $= 10 \log \frac{P_T}{P_{S(t)}} = 90$ dB

$\therefore \boxed{P_{S(t)} = 10^{-9} P_T}$



(a) for USSB Modulation :-

$(SNR)_{o/p} = (SNR)_{i/p} = 1000$ → required

$\therefore (SNR)_{i/p} = \frac{P_{S(t)}}{P_{n\,i/p}} = \frac{10^{-9} P_T}{N_0 W} = 1000$

$\therefore \frac{10^{-9} P_T}{10^{-14} * 1.5 * 10^6} = 1000 \Rightarrow \boxed{P_T = 15 \text{ kW}}$

[2] [2]

Center Share

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(b) For D.S.B.SC

$$(SNR)_{i/p} = (SNR)_{o/p} = \frac{P_{s(t)\,i/p}}{P_{n\,i/p}} = \frac{10^{-9} P_T}{N_o w} = 1000$$

$$\therefore \boxed{P_{T_{D.S.B.SC}} = 15\,kw}$$

Center Share

(c) For AM with $P = 1/3$ watt and $k_a = 0.5$

$$\therefore \frac{(SNR)_{o/p}}{(SNR)_{i/p}} = \frac{k_a^2 P}{1 + k_a^2 P}$$

$$\therefore (SNR)_{o/p} = \frac{k_a^2 P}{1 + k_a^2 P} (SNR)_{i/p}$$

$$\therefore 1000 = \frac{(0.5)^2 \frac{1}{3}}{1 + (0.5)^2 \frac{1}{3}} * \frac{P_{s(t)}}{N_o w}$$

Center Share

$$1000 = \frac{\frac{0.25}{3}}{1 + \frac{0.25}{3}} * \frac{10^{-9} P_T}{10^{-4} * 1.5 * 10^6}$$

$$\boxed{P_T = 195\,kw}$$ // [3]

[3]

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3 for S.S.B

$$S_m(f) = \begin{cases} a\frac{|f|}{w} & |f| \le w \\ 0 & 0.w \end{cases}$$

Center Share

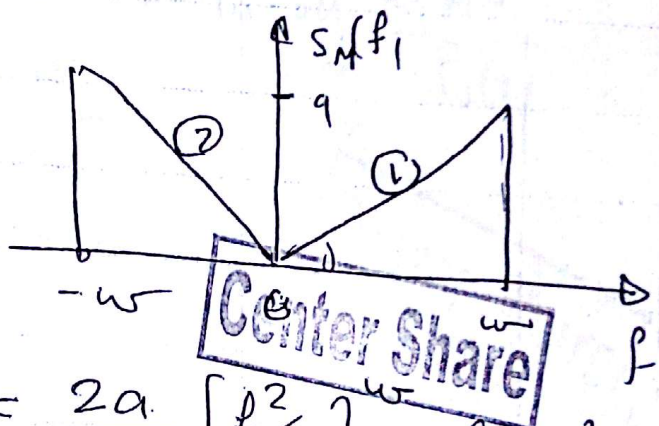
$(SNR)_{o/p} = ??$

Sol

$$\therefore (SNR)_{o/p}^{S.S.B} = \frac{A_c^2 P}{4N_0 w}$$

P is m(t) power

$$\therefore P = \int_{-w}^{w} S_M(f)\, df$$



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$$P = 2\int_0^w \frac{a}{w} f \cdot df = \frac{2a}{w}\left[f^2/2\right]_0^w = \frac{a}{w}w^2 = aw$$

$$\boxed{P = aw}$$

$$\therefore (SNR)_{o/p} = \frac{A_c^2 a w}{4 N_0 w} = \frac{A_c^2 a}{4 N_0}$$

Note

$$S_M(f) = \begin{cases} a\frac{|f|}{w} & |f| \le w \\ 0 & 0.w \end{cases} = \begin{cases} \frac{af}{w} & f \le w \quad (1) \\ -\frac{af}{w} & -f \le w = f \ge -w \quad (2) \\ 0 & \text{otherwise} \end{cases}$$

4

(4)

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④ $P_T = 40\ kw$ Channel attenuation = 80 dB

$\frac{N_0}{2} = 10^{-10}$ $W = 10^4\ Hz$

Sol

ⓐ for D.S.B

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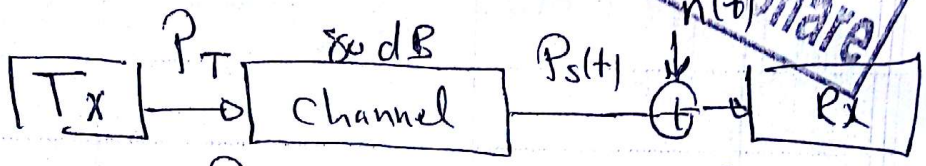

$\therefore 80\,dB = 10 \log \frac{P_T}{P_{S(t)}} = 10 \log \frac{40 * 10^3}{P_{S(t)}}$

$\therefore \boxed{P_{S(t)} = 4 * 10^{-4}\ watt}$

Center Share

$\therefore (SNR)_{i/p} = \frac{P_{S(t)}}{N_0 W} = \frac{4 * 10^{-4}}{2 * 10^{-10} * 10^4} = 2 * 10^2$

DSB-SC

$\therefore \boxed{(SNR)_{o/p} = (SNR)_{i/p} = 2 * 10^2}$

ⓑ for S.S.B

$(SNR)_{i/p} = (SNR)_{o/p} = \frac{P_{S(t)}}{N_0 W} = \frac{4 * 10^{-4}}{2 * 10^{-10} * 10^4}$

$(SNR)_{o/p} = 2 * 10^2$

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(c) For AM $\quad k_a = 0.8k$ & $P = 0.2$ watt

$$SNR(i/p) = \frac{P_{s(t)}}{N_{0W}} = 2 \times 10^2$$

But $(SNR)_0 = \dfrac{k_a^2 P}{1 + k_a^2 P} (SNR)_{i/p}$

$$(SNR)_0 = \frac{(0.85)^2 \times 0.2}{1 + (0.85)^2 \times 0.2} \times 2 \times 10^2$$

$$(SNR)_{0_{AM}} = 0.113 \times 2 \times 10^2$$

Center Share

Center Share

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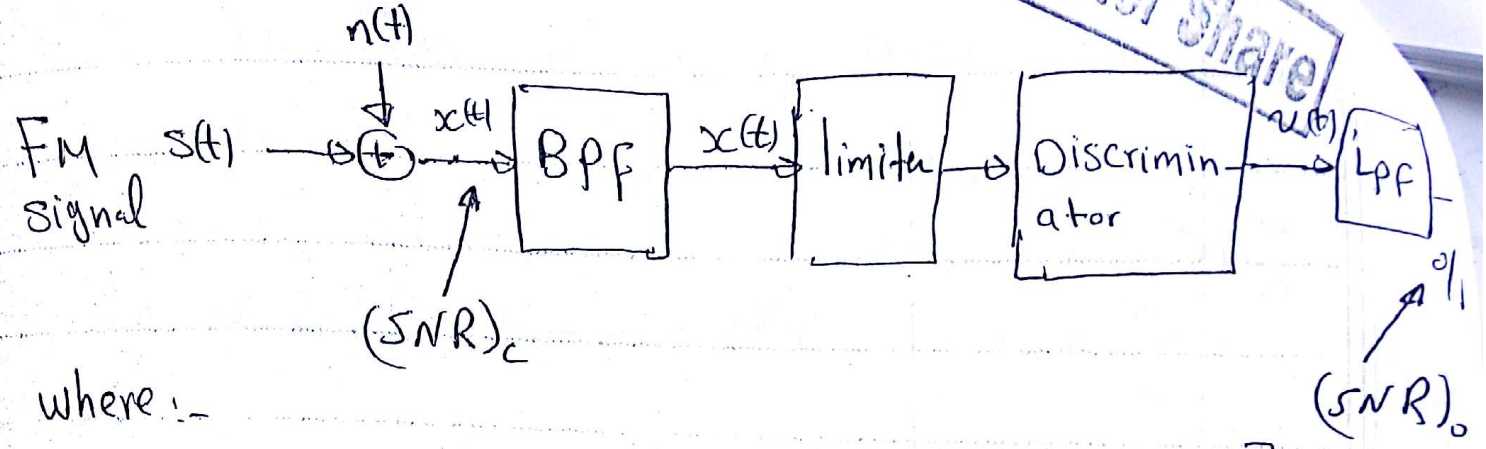
Noise In FM Receiver

Center Share



where :-

$$S(t)_{FM} = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

$$\therefore P_{s(t)} = \frac{A_c^2}{2} \quad \& \quad P_{n_{i/p}} = N_0 w$$

$$\therefore (SNR)_c = \frac{A_c^2}{2N_0 w}$$

Center Share

$(SNR)_{o/p}$

let $n(t) = n_s(t)\cos w_c t - n_q(t)\sin w_c t$

$$\therefore n(t) = r(t)\cos\left[(2\pi f_c t) + \psi(t)\right]$$

let $\phi(t) = 2\pi k_f \int_0^t m(t)\,dt$

$$\therefore x(t) = s(t) + n(t)$$

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$$x(t) = A_c \cos[(2\pi f_c t) + \phi(t)] + r(t) \cos[2\pi f_c t$$

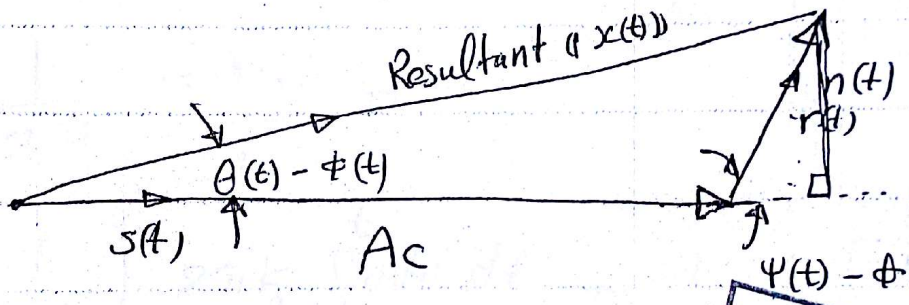
Let $x(t)$ has phase of $\theta(t)$ → المطلوب

$s(t)$ has phase $= \phi(t)$

$r(t)$ has phase $= \psi(t)$

$\psi(t)$ و $\phi(t)$ بمعلومية $\theta(t)$ المطلوب هو إيجاد

Center Share



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From the phasor diagrame.

$$\theta(t) - \phi(t) = \tan^{-1} \frac{\text{المقابل}}{\text{المجاور}} \Rightarrow \theta(t) = \phi(t) + \tan^{-1} \frac{\text{المقابل}}{\text{المجاور}}$$

$$\therefore \theta(t) = \phi(t) + \tan^{-1}\left[\frac{r(t) \sin[\psi(t) - \phi(t)]}{A_c + r(t) \cos[\psi(t) - \phi(t)]}\right]$$

$\because \tan^{-1}\alpha = \alpha$ when α is very small

$$\therefore \theta(t) = \phi(t) + \frac{r(t) \sin(\psi(t) - \phi(t)}{A_c + r(t) \cos[\psi(t) - \phi(t)]}$$

كبيرة

صغيرة بالنسبة لـ A_c

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$$\therefore \theta(t) \cong \phi(t) + \frac{r(t)\sin[\psi(t) - \phi(t)]}{Ac}$$

$$\therefore \theta(t) \cong \phi(t) + \frac{r(t)}{Ac}\sin[\psi(t) - \phi(t)]$$

$$\theta(t) \cong 2\pi k_f \int_0^t m(t).dt + \frac{r(t)}{Ac}\sin[\psi(t) - \phi(t)]$$

⇒ At Discriminator o/p :-

Center Share

$$v(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} =$$

$$= \frac{1}{2\pi}\frac{d}{dt}\left[2\pi k_f \int_0^t m(t).dt + \frac{r(t)}{Ac}\sin[\psi(t) - \phi(t)]\right]$$

$$\therefore v(t) = k_f\, m(t) + n_d(t)$$

Signal noise

Center Share

$$\therefore n_d(t) = \frac{1}{2\pi Ac}\frac{d}{dt}[r(t)\sin(\psi(t) - \phi(t))]$$

$$\therefore P_o = k_f^2 P \Longrightarrow (I)$$

$n_d(t)$ ... P_n ... ...

$$n_d(t) = \frac{1}{2\pi Ac}\frac{d}{dt}[r(t)\sin(\psi(t) - \phi(t))]$$

$$n_d(t) \cong \frac{1}{2\pi Ac}\frac{d}{dt}[r(t)\sin\psi(t)]$$

... noise ...

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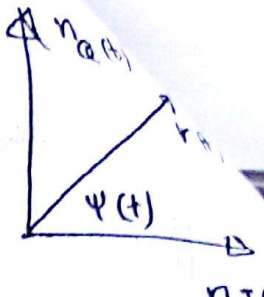
$$\therefore n_d(t) \simeq \frac{1}{2\pi A_c} \frac{d}{dt}\left[r(t)\sin\psi(t)\right]$$



But $r(t)\sin\psi(t) = n_Q(t)$

$$\therefore n_d(t) \simeq \frac{1}{2\pi A_c} \frac{dn_Q(t)}{dt}$$

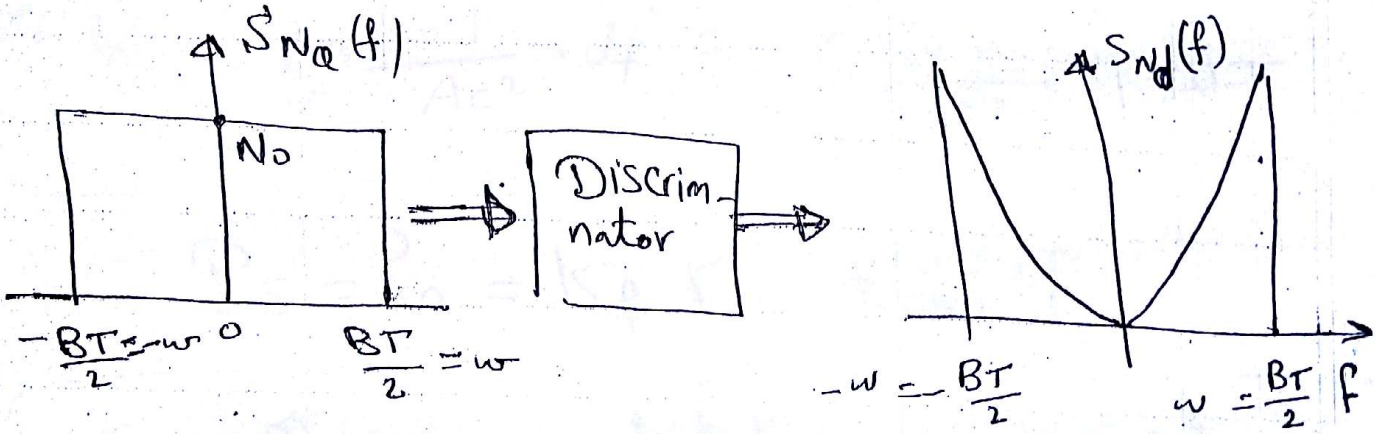
Center Share

وهذا سوف نعمل على الـ Freq Domain بدلاً من الـ time Domain وسوف نستقبل الـ التفاضل بـ $j2\pi f$ وسوف نربع ونأخذ الـ Power

$$\therefore |S_{N_d}(f)| = \left|\frac{(j2\pi f)^2}{(2\pi A_c)^2} S_{N_Q}(f)\right| = \frac{f^2}{A_c^2} S_{N_Q}(f)$$

$$\therefore S_{N_d}(f) = \frac{f^2}{A_c^2} S_{N_Q}(f)$$

Center Share



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where:

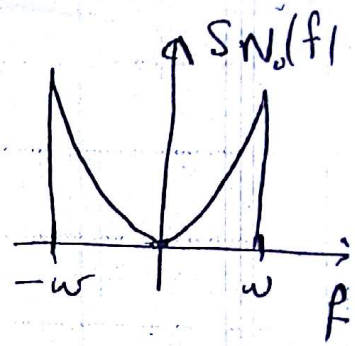
$$S_{N0}(f) = \begin{cases} N_0 & |f| \le \frac{B_T}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$S_{Nd}(f) = \frac{f^2}{A_c^2} S_{N0}(f) = \begin{cases} \frac{N_0 f^2}{A_c^2} & |f| \le \frac{B_T}{2} \\ 0 & \text{otherwise} \end{cases}$$

Center Share

After LPF with Bandwidth = w

$$\therefore S_{No}(f) = \begin{cases} \frac{N_0 f^2}{A_c^2} & |f| \le w \\ 0 & \text{otherwise} \end{cases}$$



$S_{No}(f)$ لـ $(-w \to w)$ من التكامل بـ $P_{no|p}$ وبالتالي

$$\therefore P_{n_{o|p}} = \int_{-w}^{w} \frac{N_0 f^2}{A_c^2} \cdot df = \frac{2 N_0 w^3}{3 A_c^2}$$

$$\because P_S = P_o = k_f^2 P \quad \text{from (I)}$$

$$\therefore (SNR)_{o|p} = \frac{k_f^2 P}{\frac{2 N_0 w^3}{3 A_c^2}} = \frac{3 A_c^2 k_f^2 P}{2 N_0 w^3}$$

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$$\left(\frac{S}{N}\right)_{i/p} = \frac{A_c^2}{2N_0W} \rightarrow$$

$$\therefore FOM = \frac{3A_c^2 k_f^2 P}{2N_0 W^3} \div \frac{A_c^2}{2N_0 W}$$

$$\boxed{FOM = \frac{3k_f^2 P}{W^2}} \#$$

Center Share

Ex Single tone Modulation

where

$$m(t) = A_m \cos 2\pi f_m t$$

$$\boxed{\Delta f = k_f A_m} \rightarrow \text{Peak frequency deviation}$$

$$\therefore S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

$$= A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t A_m \cos(2\pi f_m t)\,dt\right]$$

$$= A_c \cos\left[2\pi f_c t + \frac{2\pi k_f A_m}{2\pi f_m} \sin 2\pi f_m t\right]$$

$$= A_c \cos\left[2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t\right]$$

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$$s(t) = A_c \cos\left[2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t\right]$$

$\because m(t) = A_m \cos 2\pi f_m t = \frac{\Delta f}{k_f} \cos(2\pi f_m t)$

$$\therefore P = \frac{\Delta f^2}{2k_f^2} \rightarrow (1)$$

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$$\therefore (SNR)_o = \frac{3A_c^2 k_f^2 P}{2N_o w^3} = \frac{3A_c^2 k_f^2 \Delta f^2 / 2k_f^2}{2N_o w^3}$$

$$= \frac{3A_c^2 \Delta f^2}{4N_o w^3} = \frac{3A_c^2 \beta^2}{4N_o w} \quad \because \beta = \frac{\Delta f}{w}$$

Center Share

$\because \beta = \frac{\Delta f}{w}$ = Modulation index

$$(SNR)_{ch} = \frac{A_c^2}{2N_o w}$$

$$\therefore FOM = \frac{(SNR)_o}{(SNR)_c} = \frac{3}{2}\left(\frac{\Delta f}{w}\right)^2 = \frac{3}{2}\beta^2$$

↓ single tone

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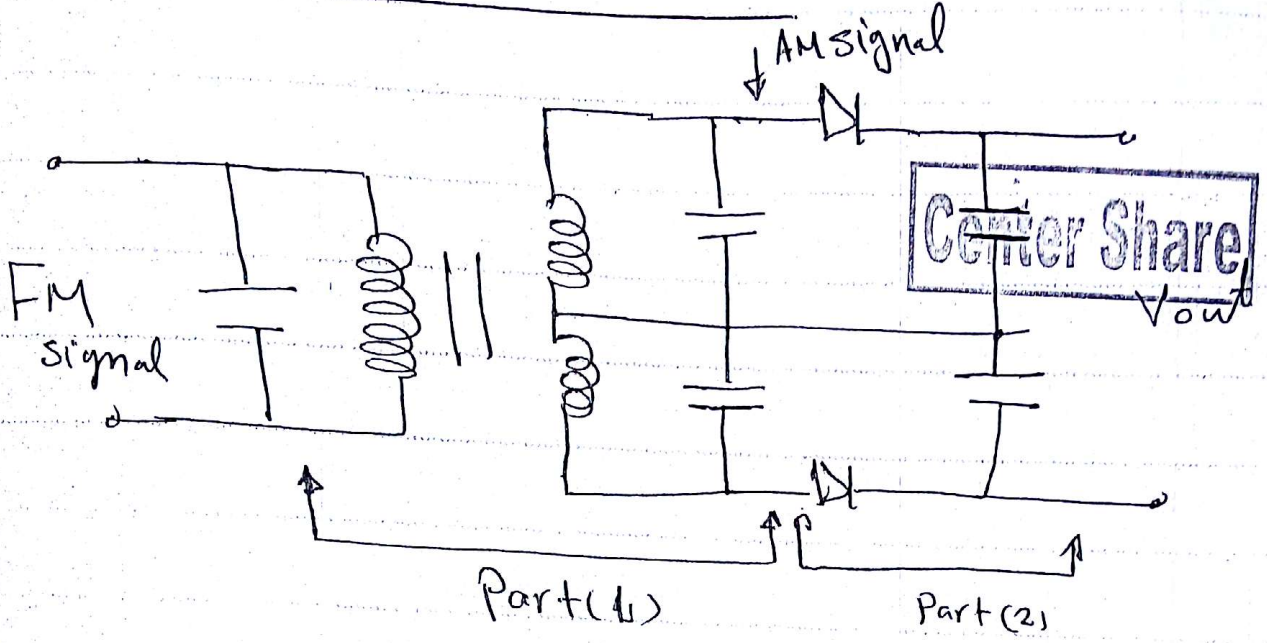
Discriminator Circuit



Center Share

وهي دائرة تتكون من جزئين

① tunned circuit

* وهي المسؤولة عن التقاط الاشارة m(t) من داخل two tunned circuits وهي عبارة عن Carrier الـ

كما انها تقوم بتحويل الاشارة من FM الى AM

② Envelope detector

* وهو المسؤول عن استرجاع اشارة m(t) من الـ AM

Center Share

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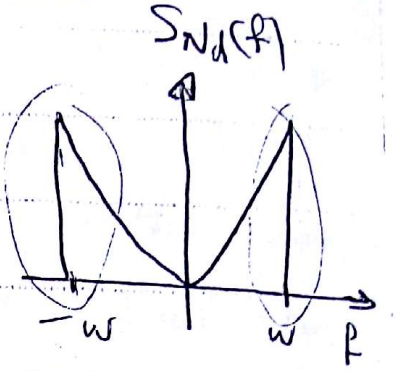
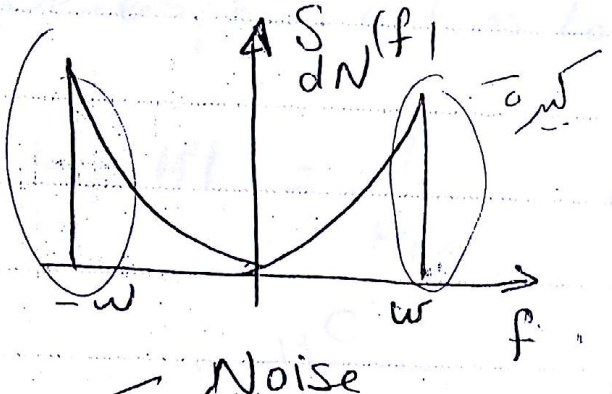
PRE-Emphasis and DE-Emphasis In FM

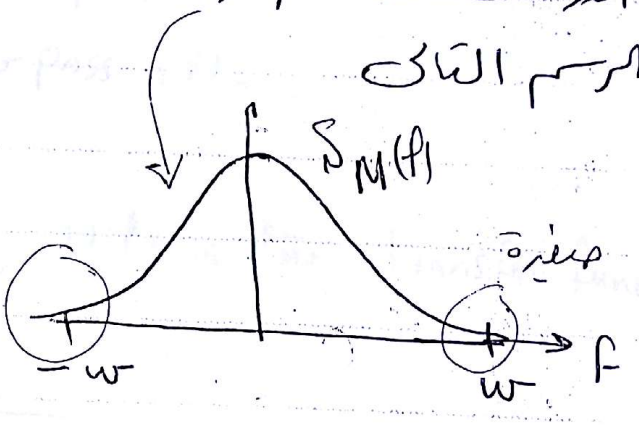
$$S_{N_d}(f) = \begin{cases} \frac{N_o f^2}{A_c^2} & |f| < w \\ 0 & \text{otherwise} \end{cases}$$



* نلاحظ أن الـ noise power مركزة بقيمة كبيرة عند $(w-w)$

* نلاحظ أن $m(t)$ وبالمقارنة بـ توزيع الـ power الخاصة بـ الـ $m(t)$ أن الـ power $m(t)$ صغيرة عند $-w < w$ كما في الرسم التالي





Noise

وبالتالي فإنه خرج الـ FM Rx سوف يتأثر بشكل كبير بـ noise عند الترددات القريبة من $w < w$ وسوف يؤدي تشوه الخرج.

* كل تلك المشكلة تم استخدام فلتر في الدخل عند الـ TX يعيد توزيع الـ power الخاصة بـ $m(t)$ على الترددات مع المحافظة على نفس كمية الـ power ليس

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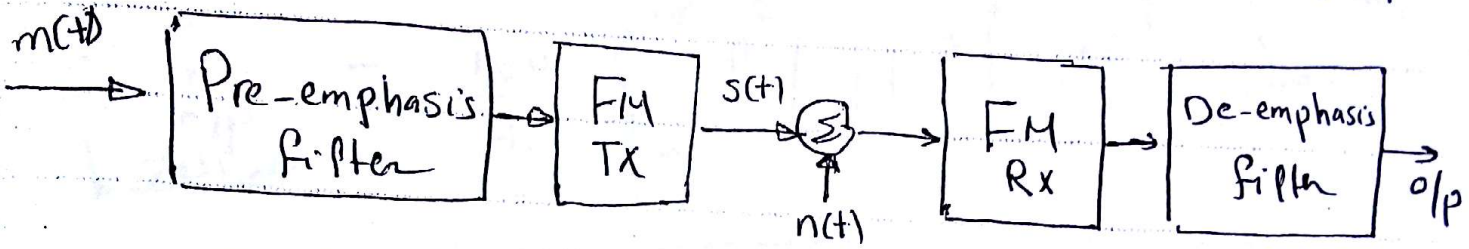
PRE - Emphasis filter.

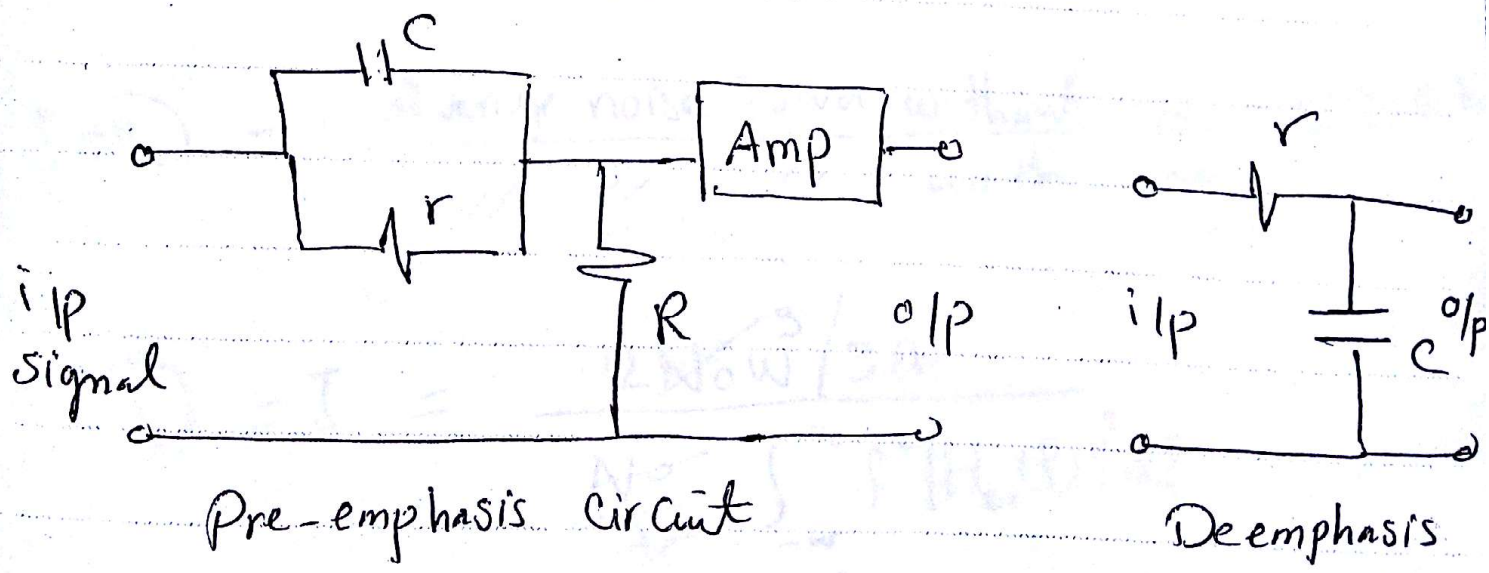
ويتم استخدام فلتر آخر في الخرج عند الـ Rx يقوم بالعملية العكسية كي تعود الـ m(t) إلى أصلها وتسمى DE-Emphasis filter as shown in the figure

بالإضافة إلى أنه سوف يقلل من قيمة الـ o/p noise



* Pre-emphasis act as high pass filter
* De-emphasis act as Low pass filter

* $|H_{pe}(f)| = \frac{1}{H_{de}(f)}$ ← H(f) is the transfer function



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Mean o/p noise after De-emphasis filter

$$\int_{-w}^{w} \frac{No}{Ac^2} f^2 df \longrightarrow \boxed{\text{De-emphasis filter}} \longrightarrow \frac{No}{Ac^2} \int_{-w}^{w} f^2 |H_{De}(f)|^2 \, df$$

$$\therefore P_{n_{o/p}} = \frac{No}{Ac^2} \int_{-w}^{w} f^2 |H_{De}(f)|^2 \, df$$ بعد

$$P_{n_{o/p}} \text{ without De-emphasis} = \int_{-w}^{w} \frac{No f^2}{Ac^2} df = \frac{2Now^3}{3Ac^2}$$ قبل

* D : is the improvement factor

$$I = D = \frac{\text{Average noise Power without DE-Emphasis filter}}{\text{// // // with // //}}$$

$$D = I = \frac{2Now^3/3Ac^2}{\frac{No}{Ac^2} \int_{-w}^{w} f^2 |H_{De}(f)|^2 df}$$

$$D = I = \frac{2w^3}{3 \int_{-w}^{w} f^2 |H_{de}(f)|^2 df}$$ بعد

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$$D = I = \frac{2w^3}{3\int_{-w}^{w} f^2 |H_{de}(f)|^2 df}$$

Ex for FM system where

$$H_{pre}(f) = 1 + \frac{jf}{f_0} \quad < \quad H_{de}(f) = \frac{1}{1 + \frac{jf}{f_0}}$$

where $f_0 = 2.1KHz$

$w = 15KHz$

find D

Sol

$$\therefore H_{de}(f) = \frac{1}{1+\frac{jf}{f_0}} \Rightarrow |H_{de}(f)| = \frac{1}{\sqrt{1+\left(\frac{f}{f_0}\right)^2}}$$

$$\therefore |H_{de}(f)|^2 = \frac{1}{1+\left(\frac{f}{f_0}\right)^2} \quad (1)$$

$$\therefore D = \frac{2w^3}{3\int_{-w}^{w} f^2 \frac{1}{1+\left(\frac{f}{f_0}\right)^2} \cdot df}$$

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$$D = \frac{2w^3}{3\int_{-w}^{w} f_0^2 \frac{f^2/f_0^2 - 1 + 1}{1+\left(\frac{f}{f_0}\right)^2} df}$$

$$= \frac{2w^3}{3\int_{-w}^{w} f_0^2 \frac{f^2/f_0^2 + 1 - 1}{1+\frac{f^2}{f_0^2}} df}$$

$$= \frac{2w^3}{3f_0^2 \int_{-w}^{w} \left(1 - \frac{1}{1+\left(\frac{f}{f_0}\right)^2}\right) df}$$

$$= \frac{2w^3}{3f_0^2 \left[2w - 2f_0 \tan^{-1}\left(\frac{w}{f_0}\right)\right]}$$

$$= \frac{2w^3}{3f_0^3 \cdot 2\left[\frac{w}{f_0} - \tan^{-1}\left(\frac{w}{f_0}\right)\right]}$$

$$= \frac{(w/f_0)^3}{3\left[\frac{w}{f_0} - \tan^{-1}\left(\frac{w}{f_0}\right)\right]}$$

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$$D = \frac{(15/2.1)^3}{3\left[\frac{15}{2.1} - \tan^{-1}\left(\frac{15}{2.1}\right)\right]} = 21$$

Radians

$$\therefore D = 10\log(21) = 13\ dB$$

Ex sheet (2)

(2)

sheet (2)

(2.) Suppose that the transfer functions of the pre-emphasis and de-emphasis filters of an FM system are scaled as follows:

$$H_{pe}(f) = k\left(1 + \frac{jf}{f_0}\right)$$

and

$$H_{de}(f) = \frac{1}{k}\left(\frac{1}{1 + jf/f_0}\right)$$

The scaling factor k is to be chosen so that the average power of the emphasized message signal is the same as that of the original message signal $m(t)$.

(a) Find the value of k that satisfies this requirement for the case when the power spectral density of the message signal $m(t)$ is

$$S_M(f) = \begin{cases} \dfrac{S_0}{1 + (f/f_0)^2}, & -W \le f \le W \\ 0, & \text{elsewhere} \end{cases}$$

(b) What is the corresponding value of the improvement factor I produced by using this pair of pre-emphasis and de-emphasis filters?

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Sol

∴ Power of $m(t)$ without pre-emphasis filter = Power of $m(t)$ with filter

$$\therefore \int_{-w}^{w} S_M(f)\cdot df = \int_{-w}^{w} S_M(f)\cdot |H_{pre}(f)|^2 \cdot df$$

$$\int_{-w}^{w} S_M(f)\,df = \int_{-w}^{w} S_M(f)\cdot k^2\left[1+\left(\frac{f}{f_0}\right)^2\right] df$$

$$\int_{-w}^{w} \frac{df}{1+\left(\frac{f}{f_0}\right)^2} = \int_{-w}^{w} k^2\,df = k^2\,2w$$

$$f_0 \tan^{-1}\frac{f}{f_0}\Big|_{-w}^{w} = 2k^2 w$$

$$2f_0 \tan^{-1}\left(\frac{w}{f_0}\right) = 2wk^2$$

$$\boxed{k = \sqrt{\frac{f_0}{w}\tan^{-1}\left(\frac{w}{f_0}\right)}} \Longrightarrow (a)$$

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$$\therefore D = \frac{2w^3}{3\int_{-w}^{w} f^2 |H_{de}(f)|^2 df}$$

$$D = \frac{2w^3}{3\int_{-w}^{w} \frac{f^2}{k^2} \frac{1}{1+\left(\frac{f}{f_0}\right)^2} df} \qquad \left|H_{de}(f)\right|^2 = \frac{1}{1+\left(\frac{f}{f_0}\right)^2}$$

$$D = \frac{2w^3}{\frac{3f_0^2}{k^2}\int_{-w}^{w} \frac{f^2/f_0^2}{1+\left(\frac{f}{f_0}\right)^2} df}$$

$$D = \frac{2w^3 k^2}{3f_0^2 \int_{-w}^{w} \frac{1+\left(\frac{f}{f_0}\right)^2 - 1}{1+\left(\frac{f}{f_0}\right)^2} df}$$

$$= \frac{2w^3 k^2}{3f_0^2 \int_{-w}^{w} \left[1 - \frac{1}{1+\left(\frac{f}{f_0}\right)^2}\right] df}$$

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$$= \frac{2\omega^3 k^2}{3f_0^2\left[2\omega - 2f_0 \tan^{-1}\left(\frac{\omega}{f_0}\right)\right]}$$

$$D = \frac{(\omega^3/f_0^3)k^2}{3\left[\frac{\omega}{f_0} - \tan\left(\frac{\omega}{f_0}\right)\right]}$$

(a) no k بالتعويض عن قيمه

$$D = \frac{\left[\omega^3/f_0^3\right]\left[\frac{f_0}{\omega}\tan^{-1}\left(\frac{\omega}{f_0}\right)\right]}{3\left[\frac{\omega}{f_0} - \tan^{-1}\left(\frac{\omega}{f_0}\right)\right]}$$

$$D = \frac{\left(\frac{\omega}{f_0}\right)^2 \tan^{-1}\left(\frac{\omega}{f_0}\right)}{3\left[\frac{\omega}{f_0} - \tan^{-1}\left(\frac{\omega}{f_0}\right)\right]}$$

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Sheet(1) (2)

Q1 Consider a phase modulation (PM) system, with the modulated wave defined by

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

where k_p is a constant and $m(t)$ is the message signal. The additive noise $n(t)$ at the phase detector input is

$$n(t) = n_I(t) \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t)$$

Assuming that the carrier-to-noise ratio at the detector input is high compared with unity, determine (a) the output signal-to-noise ratio and (b) the figure of merit of the system. Compare your results with the FM system for the case of sinusoidal modulation.

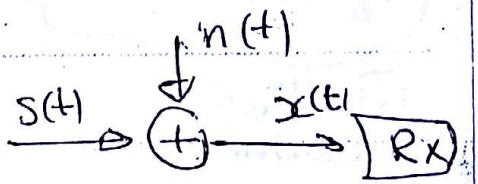
Sol

For PM system

$$S(t)_{PM} = A_c \cos[2\pi f_c t + k_p m(t)]$$

$\therefore P_{s(t)} = A_c^2/2 \quad \& \quad P_n = N_0 W$

$$\therefore (SNR)_{ch} = \frac{A_c^2}{2 N_0 W} \rightarrow (1)$$



let $\phi(t) = k_p m(t)$

let $n(t) = r(t) \cos[2\pi f_c t + \psi(t)]$

$\therefore x(t) = s(t) + n(t)$

$$x(t) = A_c \cos[2\pi f_c t + \phi(t)] + r(t) \cos[2\pi f_c t + \psi(t)]$$

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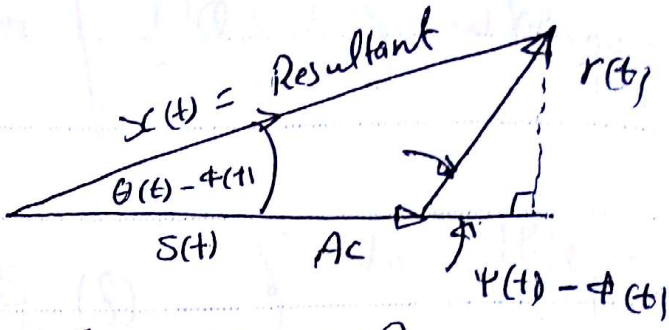
Using Phasor diagrame



$$\therefore \theta(t) = \Phi(t) + \tan^{-1} \frac{r(t)\sin(\Psi(t) - \Phi(t))}{Ac + r(t)\cos(\Psi(t) - \Phi(t))}$$

$$\theta(t) = \Phi(t) + \tan^{-1}\left[\frac{r(t)}{Ac}\sin[\Psi(t) - \Phi(t)]\right]$$

$$\theta(t) \cong \Phi(t) + \frac{r(t)\sin\Psi(t)}{Ac}$$

$\Phi(t)$: signal — $\frac{r(t)\sin\Psi(t)}{Ac}$: noise

$\tan^{-1}\alpha = \alpha$ when α is small

$$\theta(t) = k_p m(t) + \frac{n_Q(t)}{Ac}$$

$k_p m(t)$: P_o — $\frac{n_Q(t)}{Ac}$: P_n

$r(t)\sin\Psi(t) = n_Q(t)$: المركبة العمودية للـ noise

$$\therefore P_o = k_p^2 P$$

مع ملاحظة أنه كسب الـ noise power لم يكون هنا ؛ عليه تفاضل مثل ما عملت في الـ FM وتكمل حتى تأخذ الـ noise بدون

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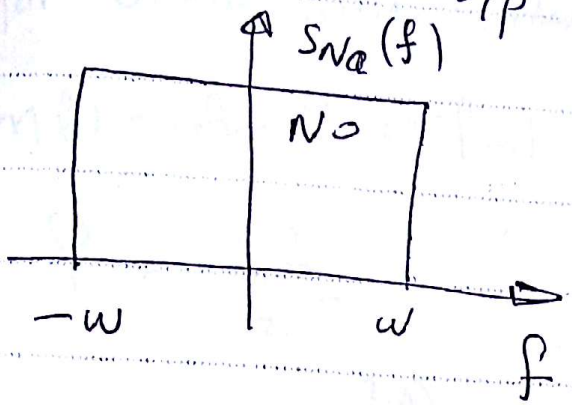
* we Calculate $P_{n_{o/p}}$ from $\left(\frac{n_Q(t)}{A_c^2}\right)$ where



$$S_{N_Q}(f) = \begin{cases} N_0 & |f| < w \\ 0 & \text{otherwise} \end{cases}$$

$$\therefore P_{n_{o/p}} = P_{n_Q} = \frac{2N_0 w}{A_c^2} \longrightarrow (2)$$

$$\therefore (SNR)_{o/p} = \frac{k_p^2 P A_c^2}{2N_0 w}$$

$$\therefore \text{FOM}_{PM} = \frac{(SNR)_{o/p}}{(SNR)_{i/p}} = \frac{k_p^2 P A_c^2 / 2N_0 w}{A_c^2 / 2N_0 w}$$

$$\text{FOM}_{PM} = k_p^2 P$$

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$\therefore$ $FOM_{PM} = k_p^2 P$

② for single tone modulation

$m(t) = A_m \cos 2\pi f_m t$

$\therefore P = \frac{A_m^2}{2}$

$\therefore FOM_{PM} = \frac{k_p^2 P}{2} = k_p^2 \frac{A_m^2}{2} = \frac{1}{2} k_p^2 A_m^2$

But $\beta_{PM} = k_p \cdot A_m$

$$\therefore FOM_{PM\ single\ tone} = \frac{1}{2} \beta_{PM}^2$$

For FM

$$FOM_{FM} = \frac{3}{2} \beta_{FM}^2$$

$\therefore FOM_{FM} = 3$ times FOM_{PM}

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Threshold Effect ← AM / FM

* وهو العيب الثاني في دوائر الـ FM < AM ويحدث عندما تكون Carrier power أعلى من الـ noise power وبالتالي فإن الـ Envelope Detector لا يستطيع إخراج الـ m(t) بشكل صحيح.

*

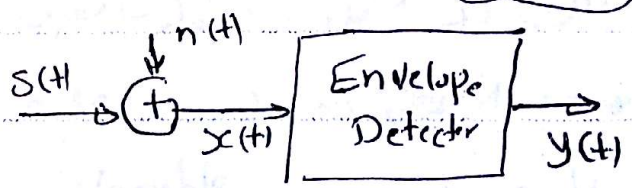
* يجب أن تكون الـ Carrier power أعلى من الـ noise power بعدد أكبر من threshold value حتى نستطيع الحصول على الإشارة m(t) بنجاح. وبالتالي سوف ندرس الحالة التي تكون فيها الـ Carrier أعلى من الـ noise

let $S(t)_{AM} = A_c[1 + k_a m(t)] \cos 2\pi f_c t$

$n(t) = r(t)\cos[2\pi f_c t + \psi(t)]$ ← (noise < Carrier)

$x(t) = S(t) + n(t)$

($S(t)$ ↓ $\frac{A_c^2}{2}$, $n(t)$ ↓ $2N_0W$)



At the envelope i/p :-

$$\text{Carrier-to-Noise ratio} = \frac{A_c^2/2}{2N_0W} = \frac{A_c^2}{4N_0W} = \rho$$

$$\therefore \rho = \frac{A_c^2}{4N_0W}$$

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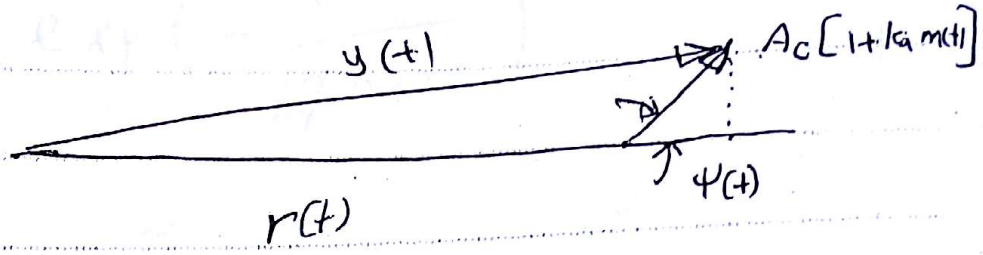
At envelop o/p : Using Phasor diagram

ع اخذ r(t) ص ال refrence بدل ص الكبيرة



$$y(t) \cong r(t) + A_c[1+k_a m(t)]\cos\psi(t) + A_c[1+k_a m(t)]\sin\psi(t)$$

$$y(t) \cong r(t) + A_c\cos\psi(t) + A_c k_a m(t)\cos\psi(t)$$

نحصل

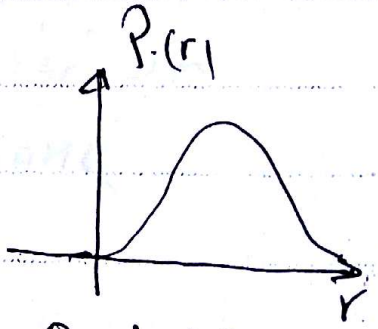
* نلاحظ أنه الخرج y(t) لا يحتوي على مركبة صريحة لل m(t) وإنما يحتوي على ال m(t) مضروبة في cosψ(t) التي تمثل ال noise وبالتالي لم نستطع الحصول على m(t) بدون ال noise

* وبالتالي يجب أن تكون ال كبيرة وأكبر من ال threshold لتفادي تلك الأشكال والطريقة المعروفة أمامنا أن $P_{Carrier}$ أكبر من P_{noise}

* مع ملاحظة أن ال noise عبارة عن متغير عشوائي يطلق عليه اسم

Probability density function of Random Variable

$$P_{Rt}(r) = \frac{r}{\sigma_N^2}\exp\left(-\frac{r^2}{2\sigma_N^2}\right)$$



Rayleigh Distribution

$$\sigma_N^2 = 2N_0W$$ Varians

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$$P_{R_t}(r) = \frac{r}{\sigma_N^2} \exp\left(-\frac{r^2}{2\sigma_N^2}\right)$$

$$P_{R_t}(r) = \frac{r}{2N_0W} \exp\left(-\frac{r^2}{4N_0W}\right)$$

and

$$P(R_t \geq A_c) = \int_{A_c}^{\infty} P_{R_t}(r)\, dr$$

probability

$$P(R_t \geq A_c) = \int_{A_c}^{\infty} \frac{r}{2N_0W} \exp\left(-\frac{r^2}{4N_0W}\right) dr$$

$$= \exp\left(-\frac{A_c^2}{4N_0W}\right) = \exp(-\rho)$$

Carrier to noise ratio

if $\rho = -1.6\,dB$ → envelop detector is well in performance

$\rho = 6.6\,dB$ → // // like Ideal Case

$\rho < -1.6\,dB$ → We Cannot restore m(t)

(Below threshold: system performance is very bad ☹)

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Ex

⇒ if $P(R_t \geq Ac) = 0.5$

$\therefore \exp(-\rho) = 0.5$

$\therefore \rho = -\ln(0.5) = 0.69$

if $P(R_t \geq Ac) = 0.01$

$\therefore \rho = -\ln(0.01) = 4.6$

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